Critical Biot Numbers of Periodic Arrays of Fins

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In this paper, we consider the heat transfer problems associated with a periodic array of triangular, longitudinal, axisymmetric, and pin fins. The problems are modeled as a wall where the flat side is isothermal and the other side, which has extended surfaces/fins, is subjected to convection with a uniform heat transfer coefficient. Hence, our analysis differs from the classical approach because (i) we consider multidimensional heat conduction and (ii) the wall on which the fins are attached is included in the analysis. The latter results in a nonisothermal temperature distribution along the base of the fin. The Biot number ($Bi = ht/k$) characterizing the heat transfer process is defined with respect to the thickness/diameter of the fins (t). Numerical results demonstrate that the fins would enhance the heat transfer rate only if the Biot number is less than a critical value, which, in general, depends on the geometrical parameters, i.e., the thickness of the wall, the length of the fins, and the period. For pin fins, similar to rectangular fins, the critical Biot number is independent of the geometry and is approximately equal to 3.1. The physical argument is that, under strong convection, a thick fin introduces an additional resistance to heat conduction.
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1 Introduction

The classical approach to estimate the heat transfer rate associated with an extended surface is to assume one-dimensional heat conduction within its boundaries and energy transfer by convection between its boundaries and the surroundings [1]. So, the conduction problem in the high conductivity material is considered, and the convection problem is simplified by modeling convection as a boundary condition to the conduction problem by assuming that the heat flux ($k\ \partial T/\partial n$) is proportional to the temperature difference between the surface (T_{surface}) and the far field (T_∞) [1], i.e.,

$$k\ \frac{\partial T}{\partial n} = h\ (T_\infty - T_{\text{surface}}) \tag{1}$$

where h is the convection heat transfer coefficient, which in general is assumed to be constant and obtained independently by analytical, numerical, and experimental methods. Furthermore, the temperature distribution along the base of the fin is assumed isothermal. Based on the one-dimensional approach, a fin effectiveness can be defined, which for the case of an infinite fin of uniform cross section is equal to [1]

$$\varepsilon_f = \sqrt{\frac{kP}{hA_c}}$$

Based on the above result, "several important trends may be inferred. Obviously, fin effectiveness is enhanced by the choice of a material of high thermal conductivity and by increasing the ratio of the perimeter (P) to the cross-sectional area (A_c). The above equation also suggests that the use of fins can better be justified under conditions for which the convection coefficient h is small" [1]. For rectangular fins, the above trends have been elucidated in Refs. [2,3].

It is established in the literature that a one-dimensional heat transfer approximation, when the underlying heat transfer process is two-dimensional, conceals some very interesting phenomena: (i) the existence of a critical thickness for planar configurations [4–6], (ii) the existence of a critical Biot number associated with a periodic array of rectangular fins [2,3], and (iii) the existence of a critical depth [5,7,8] associated with buried pipes.

In this work, similar to the work by Leontiou and Fyrillas [2,3], we extend the one-dimensional analysis by (i) assuming two-dimensional heat conduction (three-dimensional in the case of a pin fin) in the fin and (ii) by including the wall, on which the fins are attached, in the heat transfer process. The latter results in a nonisothermal temperature distribution at the base of the fin, as opposed to an isothermal distribution assumed in the classical approach. In Refs. [2,3], it is concluded that, under the limitation of a convective boundary condition with a uniform heat transfer coefficient (Eq. (1)), there exists a universal critical Biot number that determines whether a rectangular fin is effective. Unlike one-dimensional analysis [1], the critical Biot number is independent of the fin length, and it is expressed in terms of the thickness of the fin ($\text{Bi} = ht/k$). If the Biot number is larger than the critical ($\text{Bi}_{\text{crit}} = 1.64$), then the fin would attenuate the heat transfer rate. If the Biot number is less than the critical, the fin would enhance the heat transfer rate. The physical mechanism is that under strong convection, the temperature over the surface of the fin would approach the temperature of the far field and, based on Eq. (1), this would result in a reduction in convective heat transfer. Hence, under the assumption of constant convection heat transfer coefficient (Eq. (1)), an optimum fin is infinitely thin and long [1–3,9].

For the case of isothermal boundary conditions, the problem was addressed in Refs. [10–13]. If the material is a fluid, Laplace equation still applies if the fluid is under unidirectional flow and uniform temperature along the flow direction, or that the Rayleigh number is small [14,15]. In Refs. [11,12], the isoperimetric shape optimization problem of finding the optimum geometry of isothermal surfaces was addressed, while in Refs. [10] and [13] it is justified that any extension into the low conductivity material (depression) would enhance the heat transfer rate. Hence, there is a significant difference between the results associated with an optimum convective surface and an optimum isothermal surface [3].

Optimum fin design is a formidable problem in shape optimization [7,16] and, although shape optimization has been successful in obtaining the optimum shapes of buried isothermal pipes [5,17–19], isothermal surfaces [11,12], and conformal interfaces

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[20], the optimal geometries of surfaces subjected to conjugate heat transfer [21] still remain elusive. Numerical studies consider convection only to the extend that it presents a boundary condition to the conduction problem. For example, (i) parametric studies to estimate the critical Biot number of rectangular fins [3], (ii) comparison of the performance of different fins, i.e., Isachenko's optimal fin versus Schmidt's optimal fin [22], and (iii) the effect of the thermal boundary layer [7,16].

Heat transfer in extended surfaces is a fundamental problem in heat transfer [1] and is of interest to problems associated with the design of heat exchangers [23] and fins [24]. In Sec. 2, we formulate the heat transfer problem associated with periodic array of triangular, longitudinal, antisymmetric, and pin fins. In Sec. 3, the four problems are addressed numerically using the finite element method [25] to obtain the critical Biot numbers. We present our conclusions in Sec. 4.

2 Problem Formulation

Similar to Ref. [3], we will consider steady-state heat conduction in a finite slab. The heat conduction process is governed by the Laplace equation ($\nabla^2 T = 0$). The slab is bounded in one direction by an isothermal (T_0) flat surface, while the other surface is subjected to convection, with a uniform convection heat transfer coefficient (h), and a constant far-field temperature T_∞ [1], hence the boundary condition is

$$k\ \mathbf{n}\cdot\nabla T = k\ \partial T/\partial n = h\,(T - T_\infty)$$

where k is the thermal conductivity. The convective surface is not flat, rather a periodic array of fins is present (extended surfaces), in order to enhance the heat transfer rate [1,24] (Fig. 1). The case of rectangular fins was considered in Ref. [3]. In this work, we will consider four other types of classical fins: (i) triangular fins (Fig. 1(*a*)), (ii) longitudinal fins (Fig. 1(*b*)), (iii) axisymmetric fins (Fig. 1(*c*)), and (iv) pin fins (Fig. 1(*d*)). Our analysis departs from the classical approach of heat transfer from extended surfaces because we assume multidimensional heat transfer rather than one-dimensional, and we also include the effect of the wall in the heat transfer process, i.e., the base of the fin is not isothermal.

We nondimensionalize lengths with the thickness/diameter of the fins (t), and the temperature by subtracting T_0 and dividing by the temperature difference $T_\infty - T_0$. Hence, the nondimensional temperature is defined as $T = (T - T_0)/(T_\infty - T_0)$. The dimensional analysis leads to the following definition for the Biot number, $\mathrm{Bi} = h\ t/k$. The domain, the boundary conditions, and the dimensionless variables associated with the four problems are clearly indicated in Fig. 1.

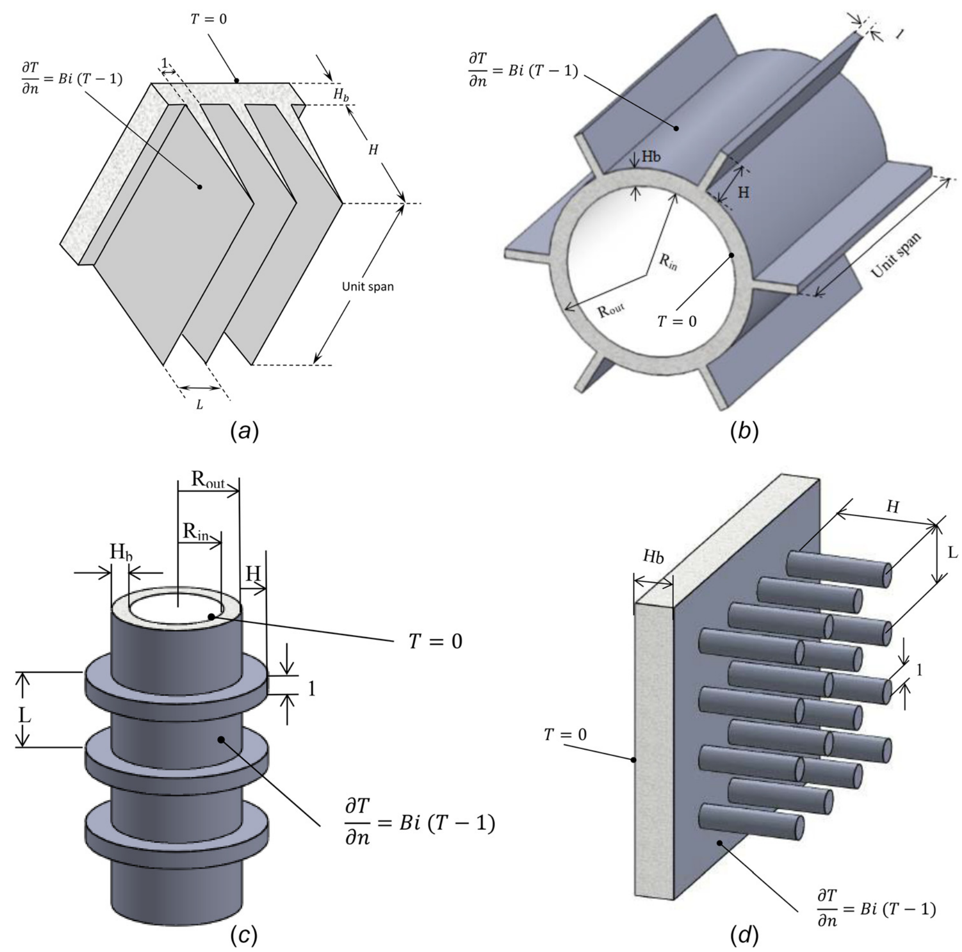

Fig. 1 Schematic representation of the four types of fins we considered. All the variables are nondimensional; lengths have been nondimensionalized with the thickness of the fins *t*, or their diameter in the case of pin fins. The (dimensionless) thickness of the wall is H_b, the (dimensionless) length of the fin is *H*, and the (dimensionless) distance between fins is *L* (period). The nondimensional temperatures are $T = 0$ at the inside surface of the wall, and $T = 1$ at the far field. The governing equation is the Laplace equation ($\nabla^2 T = 0$), i.e., conduction heat transfer, with a convection boundary condition on the outer surface: (*a*) triangular fins, (*b*) longitudinal fins, (*c*) axisymmetric fins, and (*d*) pin fins.

In Sec. 3, we will address the heat conduction problem associated with the four type of fins mentioned above. We will use finite element analysis [25] to calculate numerically the heat transfer rate and identify the critical value of the Biot numbers ($\mathrm{Bi}_{\mathrm{crit}}$), which determine when a fin is effective. A fin is effective if the heat transfer rate of the wall with a fin is larger than the heat transfer rate of a flat wall, i.e., if the Biot number is less than the critical ($\mathrm{Bi}_{\mathrm{crit}}$).

3 Critical Biot Number

In the analysis that follows, we only consider one period of the domain and we define the fin effectiveness as [2,3]

$$\varepsilon_f = \frac{q_f}{q_b} \tag{2}$$

where q_f is the dimensionless heat transfer rate of the wall with a fin obtained numerically, and q_b is the dimensionless heat transfer rate of the wall without a fin obtained using one-dimensional heat conduction analysis, i.e., a combination of thermal resistance for conduction in series with a thermal resistance for convection [1].

Hence, the definition of the fin effectiveness associated with a fin departs from the classical definition [1]. Based on the definition, an addition of an extended surface/fin would improve the heat transfer rate of the wall, if its effectiveness is greater than one [2,3]. The critical Biot number ($\mathrm{Bi}_{\mathrm{crit}}$) is defined as the Biot number where $\varepsilon_f = 1$. If for a particular fin geometry $\mathrm{Bi} < \mathrm{Bi}_{\mathrm{crit}}$, then the fin would enhance the heat transfer rate.

Besides fin effectiveness, another parameter defined in textbooks [1] is the fin efficiency. This is defined as the heat transfer rate of the fin divided by the maximum possible heat transfer rate. The latter is achieved if the entire fin surface was at the temperature of the base, i.e., if the conductivity of the material is infinite. Hence, this is realized when the $\mathrm{Bi} \to 0$.

The dimensionless heat transfer rate q_f is obtained numerically through the finite element method [25]. We used triangular elements for the two-dimensional cases (triangular, longitudinal, and axisymmetric fins), and quadrilateral elements for the three-dimensional case (pin fins) with second-order Lagrange elements as shape functions. The grid was subdivided until the difference between the heat transfer rate (q_f) of two successive refinements was less than three decimal places. Experimentation has shown that 8000 elements were required to achieve this accuracy for the two-dimensional cases, while for the three-dimensional case (pin fins) 400,000 elements were necessary. We have also taken advantage of periodicity and symmetry to truncate the domain by incorporating symmetry boundary conditions (insulation) on the appropriate boundaries. We present our results in Secs. 3.1–3.4.

3.1 Triangular Fins.

The dimensionless heat transfer rate of a wall without triangular fins (q_b) is the same as the case of rectangular fins [3], i.e., a rectangular wall of dimensions $L \times 1$ with an isothermal inner surface and convection on the outer surface (see Fig. 1(*a*))

$$q_b = \frac{\mathrm{Bi}\ L}{(1 + \mathrm{Bi}\ H_b)}$$

Extensive numerical simulations (Table 1) have shown that, within the accuracy of the numerical simulations, the critical Biot number of triangular fins depends weakly on the thickness of the base H_b and on the period L.

It strongly depends on the length of the fin H (see Fig. 2), and for long fins it approaches the value 1.64, which is the universal critical Biot number of rectangular fins [3].

Table 1 Critical Biot numbers of triangular fins for different values of the geometrical parameters. The geometrical parameters are indicated in Fig. 1(*a*).

$L/2$	H_b	H	$\mathrm{Bi}_{\mathrm{crit}}$
0.6	0.1	0.1	0.39092
0.6	0.1	0.6	1.41342
0.6	0.1	1.1	1.57499
0.6	0.1	1.6	1.60842
0.6	0.3	0.1	0.39092
0.6	0.3	0.6	1.41064
0.6	0.3	1.1	1.57021
0.6	0.3	1.6	1.62021
0.6	0.5	0.1	0.39092
0.6	0.5	0.6	1.41024
0.6	0.5	1.1	1.56942
0.6	0.5	1.6	1.60165
1.4	0.1	0.1	0.39092
1.4	0.1	0.6	1.41342
1.4	0.1	1.1	1.57499
1.4	0.1	1.6	1.60842
1.4	0.3	0.1	0.39092
1.4	0.3	0.6	1.41104
1.4	0.3	1.1	1.57071
1.4	0.3	1.6	1.60285
1.4	0.5	0.1	0.39092
1.4	0.5	0.6	1.41064
1.4	0.5	1.1	1.57021
1.4	0.5	1.6	1.60205
2.2	0.1	0.1	0.39092
2.2	0.1	0.6	1.41342
2.2	0.1	1.1	1.57499
2.2	0.1	1.6	1.60842
2.2	0.3	0.1	0.39092
2.2	0.3	0.6	1.41104
2.2	0.3	1.1	1.57081
2.2	0.3	1.6	1.60285
2.2	0.5	0.1	0.39092
2.2	0.5	0.6	1.41064
2.2	0.5	1.1	1.57021
2.2	0.5	1.6	1.60205

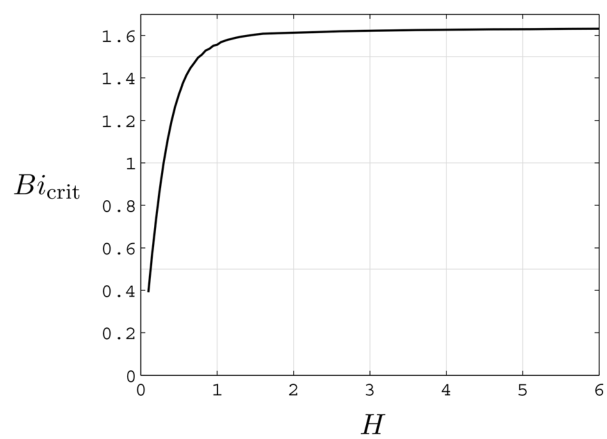

Fig. 2 Critical Biot number ($\mathrm{Bi}_{\mathrm{crit}}$) versus the length *H* of triangular fins

3.2 Longitudinal Fins.

The dimensionless heat transfer rate of a wall without longitudinal fins (q_b) is the heat transfer rate associated with a pipe of unit length, with an isothermal inner surface and convection on the outer surface (see Fig. 1(*b*)), hence

$$q_b = \frac{2\,\pi\,\mathrm{Bi}\,R_{\mathrm{out}}}{N\,(\mathrm{Bi}\,R_{\mathrm{out}}\ln[R_{\mathrm{out}}/R_{\mathrm{in}}] + 1)}$$

per unit meter, where N is the number of longitudinal fins, i.e., we have considered an $1/N$th section of the pipe because of symmetry. The only difference between a longitudinal fin and rectangular

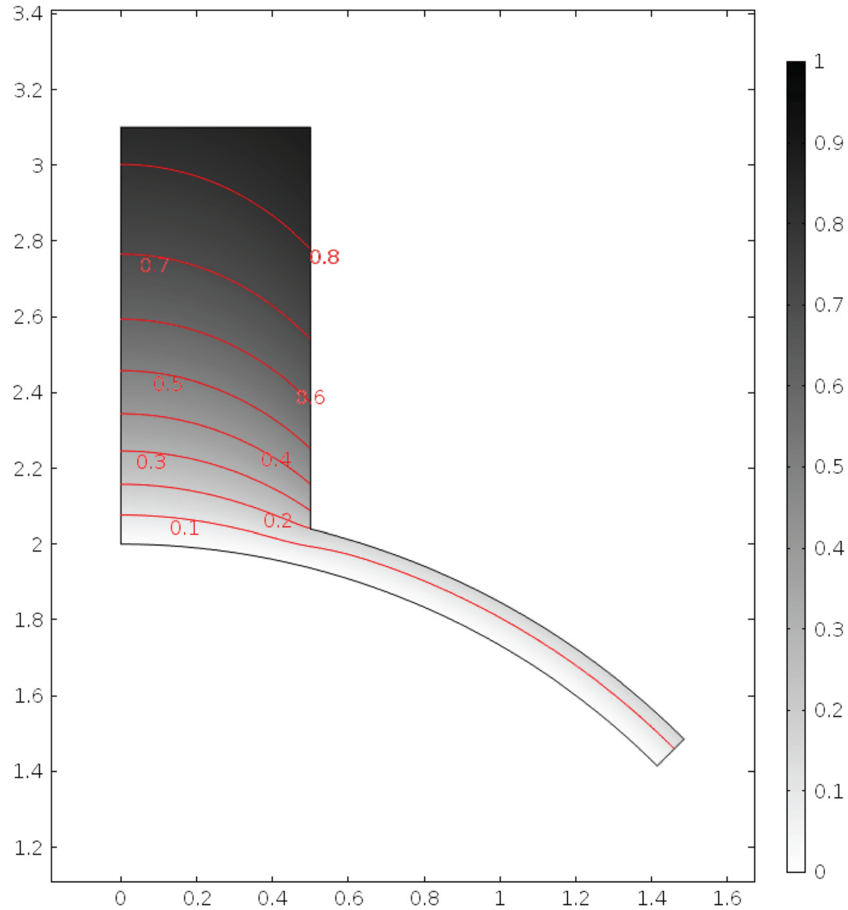

Fig. 3 A density plot of the temperature field in a tube with four longitudinal fins at critical Biot number. Higher temperatures are shown darker, as shown on the legend on the right. We also showed a number of isothermal contours 0.1, 0.2,..., 0.8. The domain and the boundary conditions are indicated in Fig. 1(*b*). We have only considered one quarter of the domain due to symmetry.

fin is the base; for a longitudinal fin, the base is a 1/Nth section of a tube, while for a rectangular fin a flat wall (Fig. 3). Hence, we also expect that the critical Biot numbers associated with longitudinal fins to be close to 1.64 [3]. Numerical simulations with combinations of geometrical parameters (Table 2) revealed that the critical Biot number decreases with increasing R_{in}, H_b, and H, and it is rather insensitive on the number of fins.

Table 2 Critical Biot numbers of longitudinal fins for different values of the geometrical parameters. The geometrical parameters are indicated in Fig. 1(*b*).

R_{in}	H_b	N	H	Bi_{crit}
3.125	1.25	6	5	1.7
3.125	1.25	6	15	1.69809
3.125	1.25	12	5	1.70407
3.125	1.25	12	15	1.70284
3.125	2.25	6	5	1.68816
3.125	2.25	6	15	1.67551
3.125	2.25	12	5	1.69036
3.125	2.25	12	15	1.68588
9.5	1.25	6	5	1.66435
9.5	1.25	6	15	1.66515
9.5	1.25	12	5	1.66444
9.5	1.25	12	15	1.66677
9.5	2.25	6	5	1.66154
9.5	2.25	6	15	1.6604
9.5	2.25	12	5	1.66204
9.5	2.25	12	15	1.66304

For the range $1 \le R_{in} \le 10$, $0.1 \le H_b \le 2$, $1 \le H \le 5$, and $2 \le N \le 10$, the critical Biot numbers lie in the range 1.65–2.1, and for an extreme case, the maximum value observed was $Bi_{crit} = 2.4$ for $R_{in} = 1$, $H_b = 0.1$, and $H = 0.1$, while they tend to 1.64 for large R_{in}, as expected. The minimum value observed was $Bi_{crit} = 1.0$ for $R_{in} = 2$, $H_b = 10$, and $H = 10$, hence any longitudinal fin would enhance the heat transfer rate if the Biot number associated with it is less than $Bi < 1$.

In Fig. 3, we show a finite element simulation of a longitudinal fin at the critical Biot number. The figure shows a density plot of the temperature field and a number of isothermal contours. The longitudinal fin is at critical Biot number ($Bi_{crit} = 1.79$), and the geometrical parameters are as follows: $H_b = 0.4$, $H = 2$, and $L = 4$. It is important to note that the base of the fin is not isothermal.

3.3 Axisymmetric Fins.

The dimensionless heat transfer rate of a flat wall without axisymmetric fins (q_b) is the heat transfer rate associated with a pipe of length L with isothermal inner surface and convection on the outer surface (see Fig. 1(*c*))

$$q_b = \frac{2\pi \, Bi \, R_{out} \, L}{(Bi \, R_{out} \ln[R_{out}/R_{in}] + 1)}$$

Numerical simulations within a range of reasonable geometrical parameters (Table 3) suggested that the critical Biot numbers lie in the range 1.45–1.75. The minimum value observed was

Table 3 Critical Biot numbers of axisymmetric fins for different values of the geometrical parameters. The geometrical parameters are indicated in Fig. 1(*c*).

$L/2$	R_{in}	H_b	H	Bi_{crit}
2.5	4.5	5.0	5.1	1.72
2.5	4.5	5.0	17.6	1.70
2.5	4.5	8.0	5.1	1.68
2.5	4.5	8.0	17.6	1.62
2.5	7.5	5.0	5.1	1.70
2.5	7.5	5.0	17.6	1.70
2.5	7.5	8.0	5.1	1.68
2.5	7.5	8.0	17.6	1.67
7.3	4.5	5.0	5.1	1.70
7.3	4.5	5.0	17.6	1.61
7.3	4.5	8.0	5.1	1.63
7.3	4.5	8.0	17.6	1.47
7.3	7.5	5.0	5.1	1.70
7.3	7.5	5.0	17.6	1.67
7.3	7.5	8.0	5.1	1.67
7.3	7.5	8.0	17.6	1.61

$Bi_{crit} = 0.44$ for the extreme case $R_{in} = 3$, $H_b = 12$, $H = 35$, and $L = 32$; hence, any axisymmetric fin would enhance the heat transfer rate if the Biot number associated with it is less than $Bi < 0.44$.

3.4 Pin Fins.

The dimensionless heat transfer rate of a wall without pin fins (q_b) is the same as the heat transfer rate of a square wall of length L with an isothermal inner surface and convection on the outer surface (see Fig. 1(*d*))

$$q_b = \frac{\text{Bi } L^2}{(1 + \text{Bi } H_b)}$$

A number of geometries have been considered (Table 4), and the conclusion is that there is a very weak dependence of the critical Biot number on the geometrical parameters. The critical Biot number is approximately 3.1.

Table 4 Critical Biot numbers of pin fins for different values of the geometrical parameters. The geometrical parameters are indicated in Fig. 1(*d*).

$L/2$	H_b	H	Bi_{crit}
1	1.5	5	3.11892
1	1.5	15	3.11774
1	1.5	20	3.1195
1	2.5	5	3.11657
1	2.5	15	3.11657
1	2.5	20	3.11716
1	3	5	3.11599
1	3	15	3.11716
1	3	20	3.11716
4	1.5	5	3.12184
4	1.5	15	3.11804
4	1.5	20	3.1195
4	2.5	5	3.11599
4	2.5	15	3.11833
4	2.5	20	3.11774
4	3	5	3.11599
4	3	15	3.11774
4	3	20	3.11482
7.5	1.5	5	3.11563
7.5	1.5	15	3.11860
7.5	1.5	20	3.11925
7.5	2.5	5	3.11554
7.5	2.5	15	3.11713
7.5	2.5	20	3.12028
7.5	3	5	3.11463
7.5	3	15	3.11824
7.5	3	20	3.11665

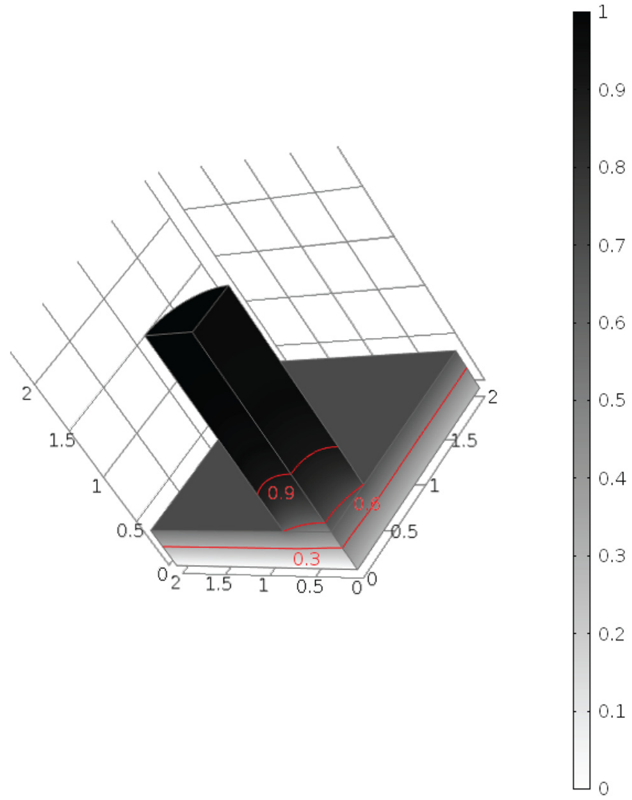

Fig. 4 A density plot of the temperature field associated with a pin fin at the critical Biot number ($Bi_{crit} = 3.1$). Higher temperatures are shown darker, as shown on the legend on the right. We also showed three isothermal contours 0.3, 0.6, and 0.9. The domain and the boundary conditions are indicated in Fig. 1(*d*). We have only considered a quarter of the domain in view of symmetries.

In Fig. 4, we show a finite element simulation of a pin fin at the critical Biot number. The figure shows a density plot of the temperature field and three isothermal contours 0.3, 0.6, and 0.9. The pin fin is at critical Biot number ($Bi_{crit} = 3.1$) and the geometrical parameters are as follows: $H_b = 0.4$, $H = 2$, and $L = 4$. It is important to note that the base of the fin is not isothermal.

4 Conclusions

The classical approach to estimate the heat transfer associated with a fin suggests that the effectiveness of a fin depends on the geometry of the fin, the conductivity of the material, and the heat transfer coefficient. In this work, we elucidate this dependence and obtain values of the critical Biot number, which determine whether a fin is effective. The problem is modeled by assuming that the heat transfer process is not one-dimensional, and that the base of the fins is not isothermal. The nonuniformity is due to the inclusion in the heat transfer process of the wall that the fins are attached to.

The geometry is that of a two-dimensional wall which is bounded in one direction by an isothermal surface and in the other direction by a periodic array of extensions/fins endowed with a uniform heat transfer coefficient. We consider triangular, longitudinal, axisymmetric, and pin fins and we determine numerically the heat transfer rate. Similar to rectangular fins considered in an earlier study, we observe that a fin would enhance the heat transfer rate only if the Biot number is less than a critical value. The Biot number is expressed in terms of the thickness/diameter of the fins, and the critical Biot number depends on the geometry. For triangular fins, the critical Biot number depends strongly only on the length of the fin and approaches 1.64 (the universal value of rectangular fins) for long fins. For longitudinal fins, the critical Biot numbers lie in the range $1.65 < \mathrm{Bi}_{\mathrm{crit}} < 2.1$ for a wide range of geometrical parameters, while for extreme cases can be as low as 1. Similar values are established for axisymmetric fields. Interestingly for pin fins, similar to rectangular fins, the critical Biot number is independent of the geometry, i.e., it is universal, and is equal to $\mathrm{Bi}_{\mathrm{crit}} = 3.1$.

The existence of a critical Biot number may be justified by realizing that the addition of a fin might present an extra resistance to heat transfer for cases of strong convection, i.e., the temperature on the surface of the fin would approach the far-field temperature, hence convective heat transfer would be reduced.

Acknowledgment

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Nomenclature

- Bi = Biot number, $\mathrm{Bi} = ht/k$
- h = heat transfer coefficient (W/(m^2 K))
- H = length of fin/extended surface (dimensionless)
- H_b = thickness of the wall (dimensionless)
- k = thermal conductivity (W/(m K))
- L = period of the geometry (dimensionless)
- N = the number of longitudinal fins
- q_b = dimensionless heat transfer rate associated with a flat wall of period L, without a fin (per unit span)
- q_f = dimensionless heat transfer rate associated with a plane wall of period L, with a fin (per unit span)
- t = thickness or diameter of fin (dimensional) length used for nondimensionalization
- T = temperature (dimensional)
- T = temperature (dimensionless, $T = (T - T_0)/(T_\infty - T_0)$)
- T_0 = temperature of the base (dimensional)
- T_∞ = temperature at the far field (dimensional)

Greek Symbol

- ε_f = fin effectiveness ($\varepsilon_f = q_f/q_b$)

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